Example 10 Finding the Domain of a Logarithmic Function

Find the domain of the function $f(x) = \ln(4 - x^2)$.

Solution As with any logarithmic function, $\ln x$ is defined when $x > 0$. Thus, the domain of f is

$$\{x \mid 4 - x^2 > 0\} = \{x \mid x^2 < 4\} = \{x \mid |x| < 2\}$$
$$= \{x \mid -2 < x < 2\} = (-2, 2)$$

■

Example 11 Drawing the Graph of a Logarithmic Function

Draw the graph of the function $y = x\ln(4 - x^2)$ and use it to find the asymptotes and local maximum and minimum values.

Solution As in Example 10 the domain of this function is the interval $(-2, 2)$, so we choose the viewing rectangle $[-3, 3]$ by $[-3, 3]$. The graph is shown in Figure 10, and from it we see that the lines $x = -2$ and $x = 2$ are vertical asymptotes.

The function has a local maximum point to the right of $x = 1$ and a local minimum point to the left of $x = -1$. By zooming in and tracing along the graph with the cursor, we find that the local maximum value is approximately 1.13 and this occurs when $x \approx 1.15$. Similarly (or by noticing that the function is odd), we find that the local minimum value is about -1.13, and it occurs when $x \approx -1.15$. ■

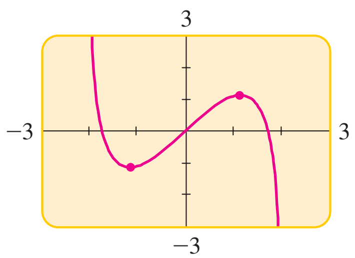


Figure 10
$y = x\ln(4 - x^2)$

4.2 Exercises

1–2 ■ Complete the table by finding the appropriate logarithmic or exponential form of the equation, as in Example 1.

1.

Logarithmic form	Exponential form
$\log_8 8 = 1$	
$\log_8 64 = 2$	
	$8^{2/3} = 4$
	$8^3 = 512$
$\log_8\left(\frac{1}{8}\right) = -1$	
	$8^{-2} = \frac{1}{64}$

2.

Logarithmic form	Exponential form
	$4^3 = 64$
$\log_4 2 = \frac{1}{2}$	
	$4^{3/2} = 8$
$\log_4\left(\frac{1}{16}\right) = -2$	
$\log_4\left(\frac{1}{2}\right) = -\frac{1}{2}$	
	$4^{-5/2} = \frac{1}{32}$

3–8 ■ Express the equation in exponential form.

3. (a) $\log_5 25 = 2$ **(b)** $\log_5 1 = 0$

4. (a) $\log_{10} 0.1 = -1$ **(b)** $\log_8 512 = 3$

5. (a) $\log_8 2 = \frac{1}{3}$ **(b)** $\log_2\left(\frac{1}{8}\right) = -3$

6. (a) $\log_3 81 = 4$ **(b)** $\log_8 4 = \frac{2}{3}$

7. (a) $\ln 5 = x$ **(b)** $\ln y = 5$

8. (a) $\ln(x + 1) = 2$ **(b)** $\ln(x - 1) = 4$

9–14 ■ Express the equation in logarithmic form.

9. (a) $5^3 = 125$ **(b)** $10^{-4} = 0.0001$

10. (a) $10^3 = 1000$ **(b)** $81^{1/2} = 9$

11. (a) $8^{-1} = \frac{1}{8}$ **(b)** $2^{-3} = \frac{1}{8}$

12. (a) $4^{-3/2} = 0.125$ **(b)** $7^3 = 343$

13. (a) $e^x = 2$ **(b)** $e^3 = y$

14. (a) $e^{x+1} = 0.5$ **(b)** $e^{0.5x} = t$

15–24 ■ Evaluate the expression.

15. (a) $\log_3 3$ **(b)** $\log_3 1$ **(c)** $\log_3 3^2$

16. (a) $\log_5 5^4$ **(b)** $\log_4 64$ **(c)** $\log_9 9$

17. **(a)** $\log_6 36$ **(b)** $\log_9 81$ **(c)** $\log_7 7^{10}$

18. **(a)** $\log_2 32$ **(b)** $\log_8 8^{17}$ **(c)** $\log_6 1$

19. **(a)** $\log_3\left(\frac{1}{27}\right)$ **(b)** $\log_{10}\sqrt{10}$ **(c)** $\log_5 0.2$

20. **(a)** $\log_5 125$ **(b)** $\log_{49} 7$ **(c)** $\log_9 \sqrt{3}$

21. **(a)** $2^{\log_2 37}$ **(b)** $3^{\log_3 8}$ **(c)** $e^{\ln\sqrt{5}}$

22. **(a)** $e^{\ln \pi}$ **(b)** $10^{\log 5}$ **(c)** $10^{\log 87}$

23. **(a)** $\log_8 0.25$ **(b)** $\ln e^4$ **(c)** $\ln(1/e)$

24. **(a)** $\log_4\sqrt{2}$ **(b)** $\log_4\left(\frac{1}{2}\right)$ **(c)** $\log_4 8$

25–32 ■ Use the definition of the logarithmic function to find x.

25. **(a)** $\log_2 x = 5$ **(b)** $\log_2 16 = x$

26. **(a)** $\log_5 x = 4$ **(b)** $\log_{10} 0.1 = x$

27. **(a)** $\log_3 243 = x$ **(b)** $\log_3 x = 3$

28. **(a)** $\log_4 2 = x$ **(b)** $\log_4 x = 2$

29. **(a)** $\log_{10} x = 2$ **(b)** $\log_5 x = 2$

30. **(a)** $\log_x 1000 = 3$ **(b)** $\log_x 25 = 2$

31. **(a)** $\log_x 16 = 4$ **(b)** $\log_x 8 = \frac{3}{2}$

32. **(a)** $\log_x 6 = \frac{1}{2}$ **(b)** $\log_x 3 = \frac{1}{3}$

33–36 ■ Use a calculator to evaluate the expression, correct to four decimal places.

33. **(a)** $\log 2$ **(b)** $\log 35.2$ **(c)** $\log\left(\frac{2}{3}\right)$

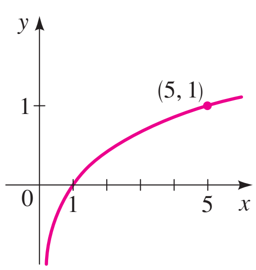
34. **(a)** $\log 50$ **(b)** $\log\sqrt{2}$ **(c)** $\log(3\sqrt{2})$

35. **(a)** $\ln 5$ **(b)** $\ln 25.3$ **(c)** $\ln(1 + \sqrt{3})$

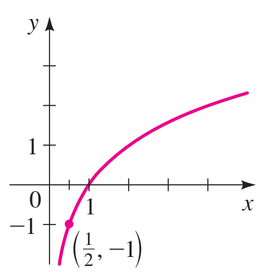
36. **(a)** $\ln 27$ **(b)** $\ln 7.39$ **(c)** $\ln 54.6$

37–40 ■ Find the function of the form $y = \log_a x$ whose graph is given.

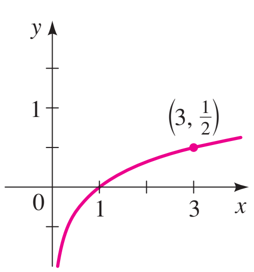
37.



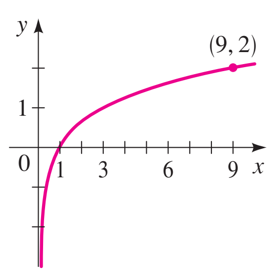
38.



39.



40.



41–46 ■ Match the logarithmic function with one of the graphs labeled I–VI.

41. $f(x) = -\ln x$

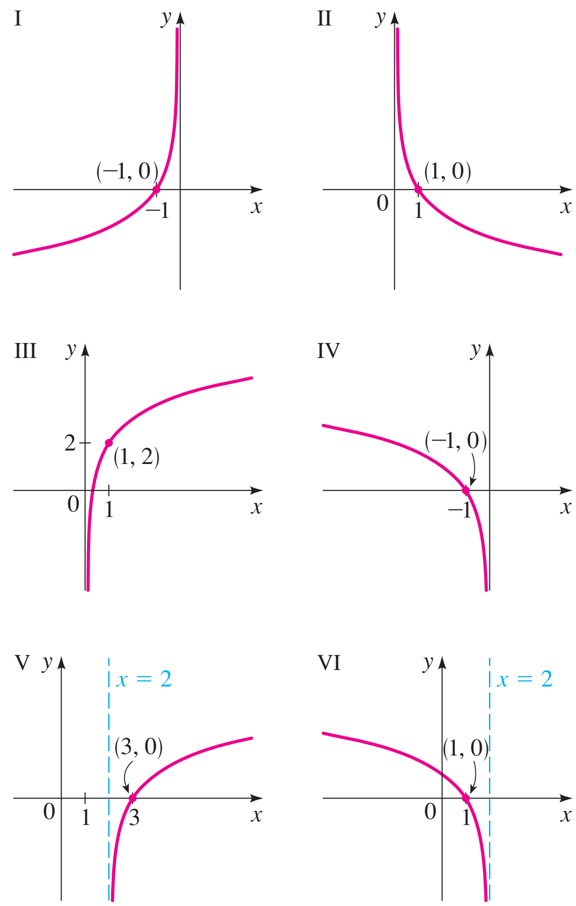
42. $f(x) = \ln(x - 2)$

43. $f(x) = 2 + \ln x$

44. $f(x) = \ln(-x)$

45. $f(x) = \ln(2 - x)$

46. $f(x) = -\ln(-x)$



47. Draw the graph of $y = 4^x$, then use it to draw the graph of $y = \log_4 x$.

48. Draw the graph of $y = 3^x$, then use it to draw the graph of $y = \log_3 x$.

49–58 ■ Graph the function, not by plotting points, but by starting from the graphs in Figures 4 and 9. State the domain, range, and asymptote.

49. $f(x) = \log_2(x - 4)$

50. $f(x) = -\log_{10} x$

51. $g(x) = \log_5(-x)$

52. $g(x) = \ln(x + 2)$

53. $y = 2 + \log_3 x$

54. $y = \log_3(x - 1) - 2$

55. $y = 1 - \log_{10} x$

56. $y = 1 + \ln(-x)$

57. $y = |\ln x|$

58. $y = \ln|x|$